Design of AAC lintels according to EN 12602

Example 1: Lintel with uniform load

1.1 Issue

Design of a lintel

Materials

AAC with a compressive strength AAC 4.5, density class 600, welded steel reinforcement with tensile yield strength 500 MPa and ultimate tensile strength 550 MPa.

EN 12602, table 1 and 2
EN 10080

1.2 Material properties

Dry Density

EN 12602, 4.2.2.3

Table 1: Density classes, dry densities in kg/m³

Density class	400	450	500	550	600	650	700
Mean dry density ρ_m	> 350 ≤ 400	> 400 ≤ 450	> 450 ≤ 500	> 500 ≤ 550	> 550 ≤ 600	> 600 ≤ 650	> 650 ≤ 700

Compressive strength

EN 12602, 4.2.4

Table 2: Compressive strength classes for AAC in MPa

Strength class	AAC 2	AAC 2.5	AAC 3	AAC 3.5	AAC 4	AAC 4.5	AAC 5
f_{ck}	2.0	2.5	3.0	3.5	4.0	4.5	5.0

1.3 Type of element

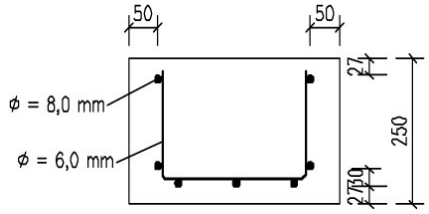


Fig. 1 Reinforcement layout of the lintel plan (dimension in mm)

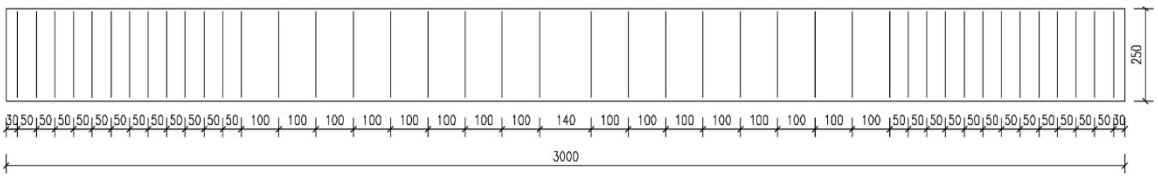


Fig. 2 Reinforcement layout of the lintel profile (dimension in mm)

2 System and dimensions

2.1 System

- dimensions: l × w × h = 3000 × 365 × 250 mm
- compressive strength class: AAC 4,5
- density class: 600 kg/m³
- support length: 250 mm (minimum support for lintels: 100 mm)
- welded steel reinforcement
    - tensile yield strength: 500 MPa
    - ultimate tensile strength: 550 MPa

Longitudinal section

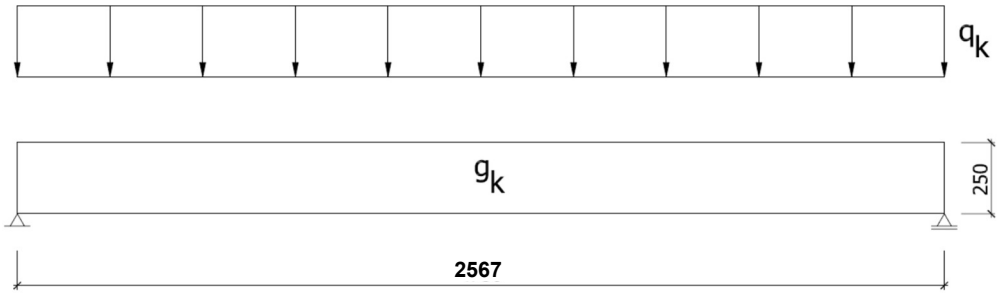


Fig. 3 Loading condition of the lintel

EN 12602, A.11

Minimum value for support length and recommended values

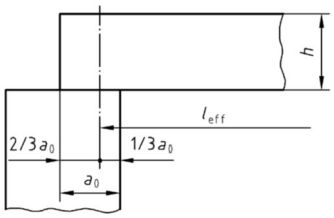


Fig. 4 Detail of the lintel support

$$l_{\text{eff}} = l_{\text{w}} - 2a_0 + \frac{1}{3}a_0 + \frac{1}{3}a_0 = 3000 - 2 \cdot 250 + \frac{1}{3} \cdot 100 + \frac{1}{3} \cdot 100 = 2567 \text{ mm}$$

The component has to be designed for all load cases also for impacts resulting from transport. The relevant load case for that is the transport with a fork lifter and for the weak axis.

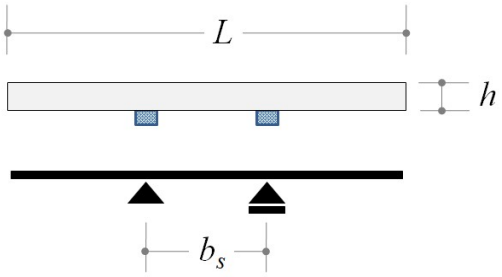


Fig. 5 Transport situation fork lift truck

Assumption for distance forks: $b_s = 1000$ mm

$l_{\text{cantilever}} = \frac{(l-b_s)}{2} = \frac{(3000-100\ )}{2} = 1000$ mm

2.2 Cross section

h = 250 mm
b = 365 mm

2.3 Concrete cover and effective depth

c_1 = 27 mm
c_2 = 27 mm

With a granted diameter of 8 mm the effective depth is:

$d = 250 - 27 - 30 \cdot \frac{2}{5} - \frac{8.0}{2} = 207.0$ mm

3 Loads

Self-weight of AAC element:
with 35 kg/m³ steel and 6 M-% moisture content of AAC
Self-weight of the lintel: 7,0 · 0,250 · 0,365 = 0,64 kN/m

EN 12602, 4.2.2.4 (1)

Load	
Self-weight of the lintel	0,64 kN/m
Permanent load[a] g_k	4.64 kN/m
Variable loads q_k	7.50 kN/m

[a]Including self-weight of the lintel.

Lintel size = 0.365 × 0.250 m

Transport weight of AAC element:
ρ_{trans} = 8.75 kN/m³

EN 12602, 4.2.2.4 (3)

4 Internal forces

Internal forces are determined for a single component with a width of 365 mm.

Load combinations acc. to EN 1990

4.1 Internal forces for characteristic combinations

$$G_{d1} = \gamma_G \cdot g_k = 1.35 \cdot 4.64 = 6.26\ \text{kN/m}$$

$$Q_{d1} = \gamma_Q \cdot q_k = 1.50 \cdot 7.50 = 11.25\ \text{kN/m}$$

$$V_{Sd1} = \frac{(G_{d1} + Q_{d1}) \cdot l_{eff}}{2} = \frac{(6.26 + 11.25) \cdot 2.567}{2} = 22.47\ \text{kN}$$

$$M_{Sd} = \frac{(G_{d1} + Q_{d1}) \cdot l_{eff}^2}{8} = \frac{(6.26 + 11.25) \cdot 2.567^2}{8} = 14.42\ \text{kNm}$$

4.2 Internal forces for frequent combinations

$$G_{d2} = G_k = 4.64\ \text{kN/m}$$

$$Q_{d2} = \psi_1 \cdot Q_k = 0.5 \cdot 7.50 = 3.75\ \text{kN/m}$$

$\psi_1 = 0.5$

$$V_{Sd2} = \frac{(G_{d2} + Q_{d2}) \cdot l_{eff}}{2} = \frac{(4.64 + 3.75) \cdot 2.567}{2} = 10.77\ \text{kN}$$

$$M_{Sd2} = \frac{(G_{d2} + Q_{d2}) \cdot l_{eff}^2}{8} = \frac{(4.64 + 3.75) \cdot 2.567^2}{8} = 6.91\ \text{kNm}$$

4.3 Internal forces for quasi-permanent combinations

$$G_{d3} = G_k = 4.64\ \text{kN/m}$$

$$Q_{d3} = \psi_2 \cdot Q_k = 0.3 \cdot 7.50 = 2.25\ \text{kN/m}$$

$\psi_2 = 0.3$

$$V_{Sd3} = \frac{(G_{d3} + Q_{d3}) \cdot l_{eff}}{2} = \frac{(4.64 + 2.25) \cdot 2.567}{2} = 8.84\ \text{kN}$$

$$M_{Sd3} = \frac{(G_{d3} + Q_{d3}) \cdot l_{eff}^2}{8} = \frac{(4.64 + 2.25) \cdot 2.567^2}{8} = 5.68\ \text{kNm}$$

4.4 Internal forces for transport situations

$G_T = \gamma_G \cdot b \cdot g_T = 1.35 \cdot 0.365 \cdot 8.75 \cdot 0.25 = 1.07\ \mathrm{kN/m}$

$V_T = \gamma_T \cdot \frac{G_T}{2} \cdot l_{cantilever} = 1.3 \cdot 1.07 \cdot 1.00 = 1.39\ \mathrm{kN}$

$M_T = \frac{\gamma_T \cdot G_T \cdot l_{cantilever}^2}{2} = \frac{1.3 \cdot 1.07 \cdot 1.00^2}{2} = 0.70\ \mathrm{kNm}$

Assumptions for transport situation:

- $\gamma_T = 1.3$ (dynamic coefficient due to manipulation of components, national regulations have to be considered)
- distance of the forks for the fork lifter:

$$1.00\ m \rightarrow l_{cantilever} = 1.00\ \mathrm{m}$$

5 Design

5.1 Material properties

Characteristic compressive strength:
$f_{ck} = 4.5\ \mathrm{MPa} = 4500\ \mathrm{kN/m^2}$

EN 12602, 4.2.4

Basic shear strength:

$\tau_{Rd} = \frac{0.063 \cdot f_{ck}^{0.5}}{\gamma_c} = \frac{0.063 \cdot 4.5^{0.5}}{1.73} = 0.0773\ \mathrm{MPa}$

EN12602, A.4.1.2 (A.6)

Mean modulus of elasticity of AAC lintel:
$E_{cm} = 5 \cdot (\rho_m - 150) = 2250\ \mathrm{N/mm^2}$

EN 12602, 4.2.7

Characteristic yield strength of steel:
$f_{yk} = 500\ \mathrm{MPa} = 500\ \mathrm{N/mm^2}$

EN 12602, 4.3.1

5.2 Design for bending

The maximum bending moment for the lintel is determined with calculating the internal lever arm h_i multiplied with the maximum permissible tension in the reinforcement $\sigma_{s,max}$.

Bottom reinforcement:

$M_{max} = h_i \cdot \sigma_{s,max}$

$$\sigma_{s,max} = \frac{A_{sl} \cdot f_{yk}}{\gamma_s} = \frac{251 \cdot 500}{1.15} = 109130 \text{ N} = 109.1 \text{ kN}$$

$$h_i = d - \frac{13}{36} \cdot x$$

Distance from top of the panel to neutral axis:

$$x = \frac{3/2 \cdot \sigma_{s,max}}{b \cdot 0.85 \cdot f_{ck}/\gamma_c} = \frac{3/2 \cdot 109130}{365 \cdot 0.85 \cdot 4.5/1.44} = 169 \text{ mm}$$

$$h_i = 207 - \frac{13}{36} \cdot 169 = 146 \text{ mm}$$

$M_{max} = 0.146 \cdot 109.1 = 15.93 \text{ kNm} > M_{Sd1} = 14.42 \text{ kNm}$

Top reinforcement:

$M_{max} = h_i \cdot \sigma_{s,max}$

$$\sigma_{s,max} = \frac{A_{sl} \cdot f_{yk}}{\gamma_s} = \frac{100.5 \cdot 500}{1.15} = 43696 \text{ N} = 43.7 \text{ kN}$$

$$h_i = d - \frac{13}{36} \cdot x$$

Distance from top of the panel to neutral axis:

$$x = \frac{3/2 \cdot \sigma_{s,max}}{b \cdot 0.85 \cdot f_{ck}/\gamma_c} = \frac{3/2 \cdot 43696}{365 \cdot 0.85 \cdot 4.5/1.44} = 68 \text{ mm}$$

$$h_i = 219 - \frac{13}{36} \cdot 68 = 194 \text{ mm}$$

$M_{max} = 0.194 \cdot 43.7 = 8.48 \text{ kNm} > M_T = 0.70 \text{ kNm}$

5.3 Minimum reinforcement

EN 12602, A.3.4 (A.3)

$f_{cflm} = 0.27 \cdot 4.5 = 1.215 \text{ MPa}$

$A_{ct} = b \cdot h/2 = 36.5 \cdot 12.5 = 456 \text{ cm}^2$

$A_{s,min} = k \cdot A_{ct} \cdot f_{cflm} / f_{yk} = 0.4 \cdot 456 \cdot 1.215 / 500 = 0.44 \text{ cm}^2$

$A_{s,min} = 0.44 \text{ cm}^2 < 2.51 \text{ cm}^2$

5.4 Design for shear force

EN 12602, A.4

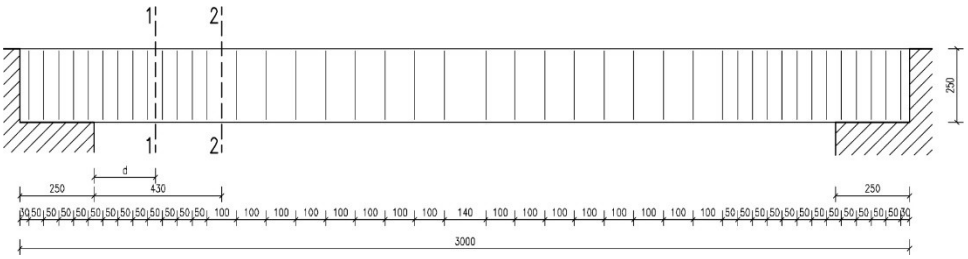


Fig. 6 Reinforcement layout of the lintel profile (dimension in mm)

Determination of reinforcement ratio:

$$\rho_l = \frac{A_{s,exis}}{b \cdot d} = \frac{2.51}{36.5 \cdot 20.7} = 0.0033 < 0.005$$

Minimum design value of shear force **(design shear resistance of the lintel without reinforcement):**

EN 12602, (A.6)

$$V_{Rd1} \geq 0.06 \cdot \frac{f_{ck}}{\gamma_c} \cdot b_w \cdot d = 0.06 \cdot \frac{4500}{1.73} \cdot 0.365 \cdot 0.207 = 11.79 \text{ kN}$$

$$V_{Rd1} = \tau_{Rd} \cdot (1 - 0.83 \cdot d) \cdot (1 + 240 \cdot \rho_l) \cdot b_w \cdot d = 77.3 \cdot (1 - 0.83 \cdot 0.207) \cdot (1 + 240 \cdot 0.0033) \cdot 0.365 \cdot 0.207 = 8.67 \text{ kN}$$

Higher value is decisive: $V_{Rd1} = 11.79$ kN

Shear resistance V_{Rd2} (maximum design shear capacity):

EN 12602, 5.2.7.2.2

$$V_{Rd2} = b_w \cdot z \cdot \upsilon \cdot f_{cd} \cdot \left(\frac{\cot\theta + \cot\alpha_s}{1 + \cot^2\theta}\right) = 0.365 \cdot 0.176 \cdot 0.56 \cdot \frac{4500}{1.73} \cdot \left(\frac{1+0}{1+1}\right) = 46.8 \text{ kN}$$

$V_{Rd2} = 46.8$ kN > 22.47 kN = V_{Sd1}

Section 1-1

$$V_{Sd1\text{-}1} = V_{Sd1} - \left(\frac{100}{3} + d\right) \cdot (G_{d1} + Q_{d1}) = 22.47 - \left(\frac{100}{3} + 207\right)/1000 \cdot (6.26 + 11.25)$$

$V_{Sd1\text{-}1} = 18.26$ kN > $V_{Rd1} = 11.79$ kN $\Rightarrow$ shear reinforcement required

$$V_{Rd3} = V_{Rd1} + V_{wd}$$

$$V_{wd} = \frac{A_{sw}}{s} \cdot z \cdot \sigma_{swd}$$

$$\sigma_{swd} = 0.45 \cdot K_1 \cdot K_2 \cdot f_{c0} \cdot \frac{\emptyset_{sl}^2 + (K_3 \cdot \emptyset_{sw})^2}{\emptyset_{sw}^2} \leq \frac{f_{ywk}}{\gamma_s} \leq 200 \text{ MPa}$$

$$K_1 = 0.33 \cdot \frac{e}{\emptyset_{sl}} = 0.33 \cdot \frac{50}{8} = 2.06 \geq 1 \Rightarrow K_1 = 1$$

$K_2 = \frac{s}{\emptyset_{sl}} = \frac{50}{8} = 6.25 \leq 14 \Rightarrow K_2 = 6.25$

open stirrups $\Rightarrow K_3 = 0$

$\sigma_{swd} = 0.45 \cdot 1 \cdot 6.25 \cdot 2.4 \cdot \frac{4.5}{1.73} \cdot \frac{8.0^2}{6.0^2} = 31.2 \text{ MPa} \leq \frac{500}{1.15} = 434.8 \leq 200 \text{ MPa}$

$\Rightarrow \sigma_{swd} = 31.2 \text{ MPa}$

$V_{wd} = \frac{A_{sw}}{s} \cdot z \cdot \sigma_{swd} = \frac{2 \cdot 28.3}{50} \cdot 0.9 \cdot 207 \cdot 31.2 = 6579 \text{ N} = 6.58 \text{ kN}$

$V_{Rd3} = V_{Rd1} + V_{wd} = 11.79 + 6.58 = 18.37 \text{ kN} > V_{Sd1\text{-}1} = 18.26 \text{ kN}$

Section 2-2

$V_{Sd2\text{-}2} = V_{Sd1} - \left(\frac{100}{3} + 430\right) \cdot (G_{d1} + Q_{d1}) = 22.47 - \left(\frac{100}{3} + 430\right)/1000 \cdot (6.26 + 11.25)$

$V_{Sd2\text{-}2} = 14.36 \text{ kN}$

$V_{Rd1} = 11.79 \text{ kN} < 14.36 \text{ kN} = V_{Sd2\text{-}2} \Rightarrow$ shear reinforcement required

$V_{Rd3} = V_{Rd1} + V_{wd}$

$V_{wd} = \frac{A_{sw}}{s} \cdot z \cdot \sigma_{swd}$

$\sigma_{swd} = 0.45 \cdot K_1 \cdot K_2 \cdot f_{c0} \cdot \frac{\emptyset_{sl}^2 + (K_3 \cdot \emptyset_{sw})^2}{\emptyset_{sw}^2} \leq \frac{f_{ywk}}{\gamma_S} \leq 200 \text{ MPa}$

$K_1 = 0.33 \cdot \frac{e}{\emptyset_{sl}} = 0.33 \cdot \frac{50}{8} = 2.06 \geq 1 \Rightarrow K_1 = 1$

$K_2 = \frac{s}{\emptyset_{sl}} = \frac{100}{8} = 12.5 \leq 14 \Rightarrow K_2 = 12.5$

open stirrups $\Rightarrow K_3 = 0$

$\sigma_{swd} = 0.45 \cdot 1 \cdot 12.5 \cdot 2.4 \cdot \frac{4.5}{1.73} \cdot \frac{8.0^2}{6.0^2} = 62.4 \text{ MPa} \leq \frac{500}{1.15} = 434.8 \leq 200 \text{ MPa}$

$\Rightarrow \sigma_{swd} = 62.4 \text{ MPa}$

$V_{wd} = \frac{A_{sw}}{s} \cdot z \cdot \sigma_{swd} = \frac{2 \cdot 28.3}{100} \cdot 0.9 \cdot 207 \cdot 62.4 = 6579 \text{ N} = 6.58 \text{ kN}$

$V_{Rd3} = V_{Rd1} + V_{wd} = 11.79 + 6.58 = 18.37 \text{ kN} > V_{Sd2\text{-}2} = 14.36 \text{ kN}$

6 Anchoring of longitudinal reinforcement

EN 12602, A.10.3

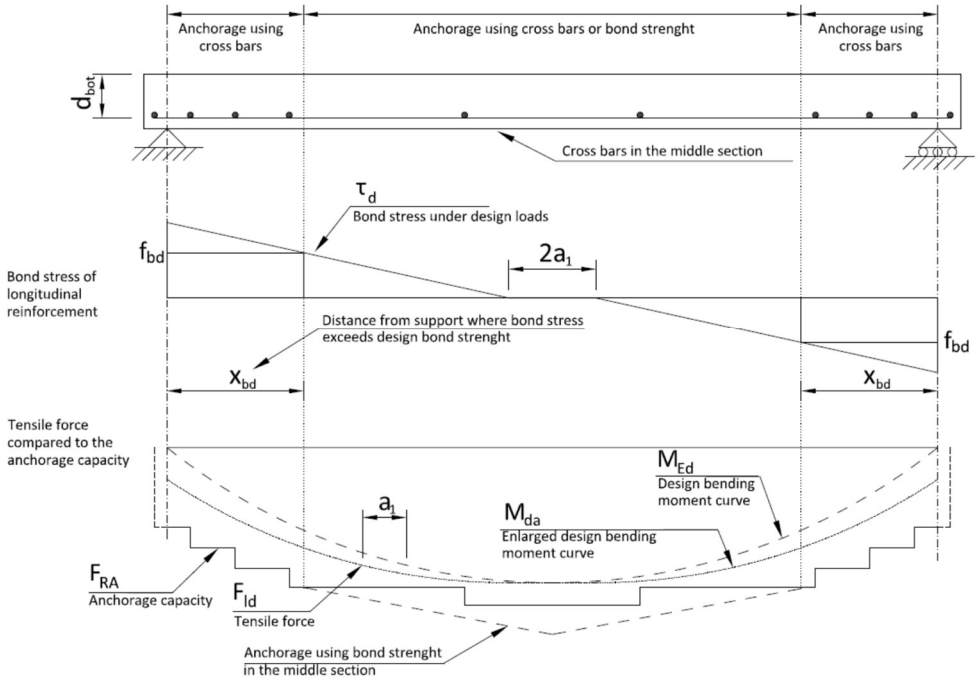
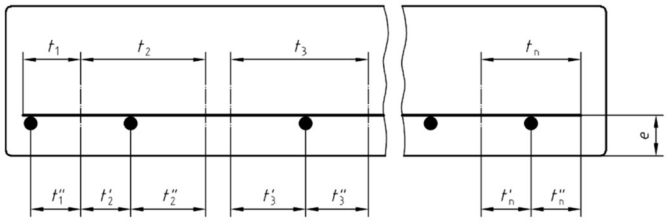


Fig. 7 Principle of anchorage



Cross-sectional view

Fig. 8 Effective length of transverse anchorage bars

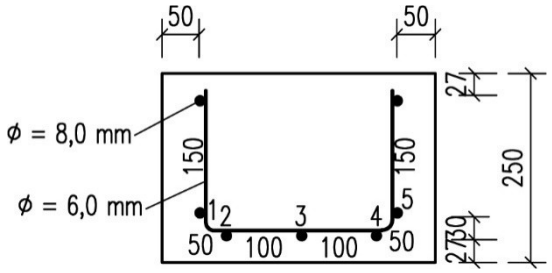


Fig. 9 Cross section of the lintel with distances between the longitudinal bars

Description reinforcement layout:

$\emptyset_{sl}$ = 8.0 mm

$\emptyset_t$ = 6.0 mm

s_{l1} = 50 mm and 100 mm (see Fig. 9)

distance to bottom side of panel:

$e = c + \emptyset_{sl} + \emptyset_{sw}/2 = 27 + 8.0 + 6.0/2 = 38.0$ mm

Effective length of transverse bars (see):

$t_i', t_i'' \leq 8 \cdot \emptyset_t = 48$ mm, $t_i' + t_i'' \leq 14 \cdot \emptyset_t = 84$ mm

$t_1 = t_5 = t_1' + t_1'' = t_5' + t_5'' = 48 + 25$ mm $= 73$ mm $< 14 \cdot \emptyset_t = 84$ mm

$\Rightarrow t_1 = t_5 = 73$ mm

$t_2 = t_4 = t_2' + t_2'' = t_4' + t_4''48 + 25$ mm $= 73$ mm $< 14 \cdot \emptyset_t = 84$ mm

$\Rightarrow t_2 = t_4 = 73$ mm

$t_3 = t_3' + t_3'' = 48 + 48$ mm $= 96$ mm $> 14 \cdot \emptyset_t = 84$ mm

$\Rightarrow t_3 = 84$ mm

$t_t = t_1 + t_2 + t_3 + t_4 + t_5 = 73 + 73 + 84 + 73 + 73$ mm $= 376$ mm

Maximum tensile force:

$F_{ld,max} = M_{Sd1} / z = 14.42 / (0.9 \cdot 0.207) = 77.40$ kN

$F_{ld,support} = M_{Sd1,support} / z = 4.50 / (0.9 \cdot 0.207) = 24.15$ kN

Design value for bearing strength at support (m = 1.3; n_p = 5, transverse compression at support):

$$f_{ld,support} = \frac{1.35 \cdot m \cdot (e/\emptyset_t)^{1/3} \cdot \alpha \cdot f_{ck}}{\gamma_c} \leq 2.2 \cdot \frac{f_{ck}}{\gamma_c}$$

$$f_{ld,support} = \frac{1.35 \cdot 1.3 \cdot (38.0/6.0)^{1/3} \cdot 0.85 \cdot 4.5}{1.44} \leq 2.2 \cdot \frac{4.5}{1.44} = 8.62 \text{ MPa} < 6.88 \text{ MPa}$$

$\Rightarrow f_{ld,support} = 6.88$ MPa

Design value for bearing strength at middle of span (m = 1.071; n_p = 5, transverse compression at support):

$$f_{ld,field} = \frac{1.35 \cdot m \cdot (e/\emptyset_t)^{1/3} \cdot \alpha \cdot f_{ck}}{\gamma_c} \leq 2.2 \cdot \frac{f_{ck}}{\gamma_c}$$

$$f_{ld,support} = \frac{1.35 \cdot 1.071 \cdot (38.0/6.0)^{1/3} \cdot 0.85 \cdot 4.5}{1.73} \leq 2.2 \cdot \frac{4.5}{1.73} = 6.75 \text{ MPa} > 5.72 \text{ MPa}$$

$\Rightarrow f_{ld,field} = 5.72$ MPa

Bond Class B1

where, α is a reduction coefficient for long term effect on compressive strength of AAC (α = 0.85)

EN 12602, A.3.2

Anchorage force capacity (F_{RA}):

$$F_{RA,support} = 0.83 \cdot n_t \cdot \emptyset_t \cdot t_t \cdot f_{ld,support} \leq 0.6 \cdot n_l \cdot n_t \cdot F_{wg} / \gamma_s$$
$$= 0.83 \cdot 5 \cdot 6.0 \cdot 376 \cdot 6.88 \leq 0.6 \cdot 5 \cdot 5 \cdot 0.25 \cdot A_{sl} \cdot f_{yk} / \gamma_s$$
$$= 64.41 \text{ kN} < 81.95 \text{ kN}$$

Welding Strength Class S1

$$F_{RA,max} = \sum min \left[0.83 \cdot \varphi_{tot} \cdot t_t \cdot f_{ld}(n_t); \frac{0.60 \cdot n_l \cdot n_t \cdot F_{wg}}{\gamma_s} \right] = 220.8 \text{ kN} < 448.9 \text{ kN}$$

$F_{RA,support}$ = 64.41 kN > $F_{ld,support}$ = 12.01 kN and
$F_{RA,max}$ = 220.8 kN > $F_{ld,max}$ = 77.40 kN

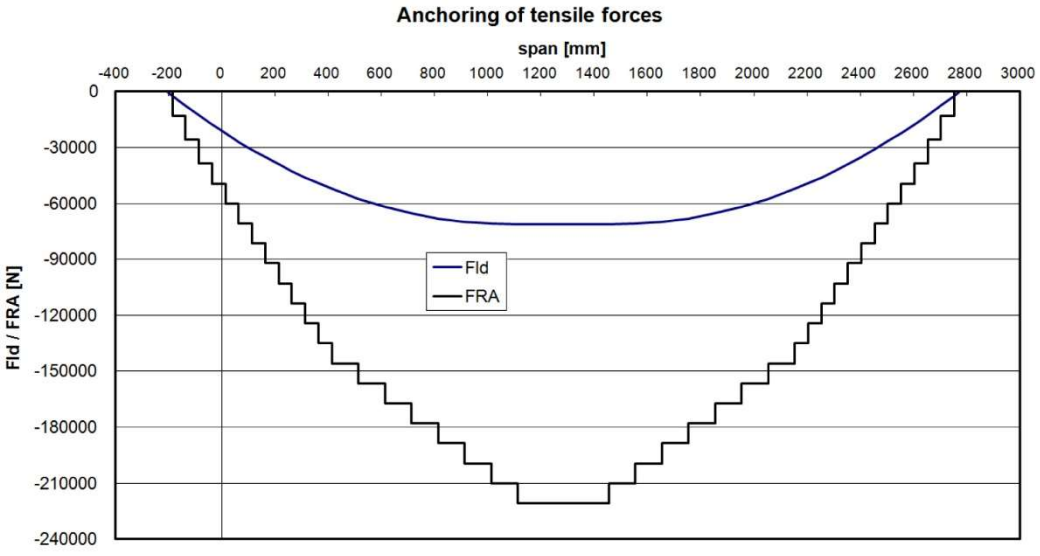


Fig. 10 Comparison of anchorage capacity and design tensile force

The figure above shows that the anchorage capacity force does exceed the design tensile force at each section of the lintel.

As we can see from Fig. 2 that the anchorage capacity force does exceed the design tensile force at each section of the panel.

Therefore, the assumption is satisfied for the required conditions.

The sufficient anchorage has to be proven also for the upper reinforcement (according to the method above) which is not shown here.

7 Serviceability Limit States

EN 12602, A.9.4

Cracking moment:
$M_{cr} = (b \cdot h^2 / 6) \cdot f_{cflm} = (0.365 \cdot 0.25^2 / 6) \cdot (0.27 \cdot 0.8 \cdot 4500) = 3.70$ kNm

EN 12602, A.9.4.3 and 4.2.5

$f_{cflm} = 0.27 \cdot 0.8 \cdot f_{ck}$

$M_{Sd1} > M_{cr}$, so the lintel is in an intermediate situation between uncracked and cracked condition

General note:
The limit value for the maximum deflection may be found in a national application document. The recommended value for the calculated deflection of roof and floor components subjected to quasi-permanent loads is (according to EN 12602) span/250.

EN 12602, (A.42)

7.1 Deflection under uncracked condition

7.1.1 Short-term deflection

Ratio of modulus of elasticity of reinforcing steel and AAC:
$n = \frac{E_s}{E_{cm}} = \frac{200000}{2250} = 88.9$
Moment of area of AAC and reinforcement:
$I_{c,brutto} = \frac{b \cdot h^3}{12} + n \cdot (5 \cdot \pi \cdot (\emptyset_1/2)^4/4 + 2 \cdot \pi \cdot (\emptyset_2/2)^4/4) = 47538.6$ cm^4

Parts of moment of inertia from consideration of the reinforcement:
$A_{s1} = 2.51$ cm^2, $A_{s2} = 1.01$ cm^2

Centre of gravity:
$y_S = \frac{b \cdot h \cdot h/2 + n \cdot (A_{s1} \cdot y_{s1} + A_{s2} \cdot y_{s2})}{b \cdot h + n \cdot (A_{s1} + A_{s2})} = \frac{14332.1}{1225.4} = 11.70$ cm
y_{s1} and y_{s2} are the distances from the centre of the reinforcement steel to the bottom surface of the lintel.

$I_{ST} = b \cdot h \cdot (\frac{h}{2} - y_s)^2 + n \cdot (A_{s1} \cdot (y_{s1} - y_s)^2 + A_{s2} \cdot (y_{s2} - y_s)^2)$
$= 36.5 \cdot 25.0 \cdot (12.5 - 11.7)^2 + 88.9 \cdot (2.51 \cdot (4.3 - 11.7)^2 + 0.85 \cdot (21.9 - 11.7)^2) = 20664.9$ cm^4

$E_{cm} \cdot I_{ci} = E_{cm} \cdot (I_{C;BRUTTO} + I_{st}) = 2250 \cdot (47638.6 + 20664.9) \cdot 10^{-8} = 1.535$ MN/m^2

Deflection due to load combination 2 (frequent action combinations):

$y_{el} = \frac{5}{48} \cdot \frac{M_{Sd2} \cdot l_{eff}^2}{E_{cm} \cdot I_{ci}} = \frac{5}{48} \cdot \frac{0.00691 \cdot 2.567^2}{1.535} = 0.00309$ m

$y_{el} = 0.00309$ m = 0.31 cm < 1.03 cm = $l_{eff} / 250$

EN 12602, 9.4.1, Note 1

7.1.2 Long-term deflection

EN 12602, (A.43)

Effective modulus of elasticity:

$E_{c,eff} = E_{cm} / (1 + \varphi) = 2250 / (1 + 1) = 1125$ N/mm^2

Ratio of modulus of elasticity of reinforcing steel and AAC:

$n = \frac{E_s}{E_{c,eff}} = \frac{200000}{1125} = 177.8$

Moment of area of AAC and reinforcement:

$I_{c,brutto} = \frac{b \cdot h^3}{12} + n \cdot (5 \cdot \pi \cdot (\emptyset_1/2)^4/4 + 5 \cdot \pi \cdot (\emptyset_2/2)^4/4) = 47551.1$ cm^4

Centre of gravity:

$y_S = \frac{b \cdot h \cdot h/2 + n \cdot (A_{s1} \cdot y_{s1} + A_{s2} \cdot y_{s2})}{b \cdot h + n \cdot (A_{s1} + A_{s2})} = \frac{17258.0}{1538.4} = 11.22$ cm

Moment of inertia for reinforcement:

$I_{ST} = b \cdot h \cdot (\frac{h}{2} - y_s)^2 + n \cdot (A_{s1} \cdot (y_{s1} - y_s)^2 + A_{s2} \cdot (y_{s2} - y_s)^2) = 36.5 \cdot 25.0 \cdot (12.5 - 11.22)^2 + 177.8 \cdot (2.51 \cdot (4.3 - 11.22)^2 + 1.01 \cdot (21.9 - 11.22)^2) = 43348.8$ cm^4

$E_{c,eff} \cdot I_{ci} = E_{c,eff} \cdot (I_{C;BRUTTO} + I_{st}) = 1125 \cdot (47551.1 + 43348.8) \cdot 10^{-8} = 1.023$ MN/m^2

Deflection due to load combination 3 (quasi-permanent combinations):

$y_\infty = \frac{5}{48} \cdot \frac{M_{Sd3} \cdot l_{eff}^2}{E_{c,eff} \cdot I_{ci}} = \frac{5}{48} \cdot \frac{0.00568 \cdot 2.567^2}{1.023} = 0.00381$ m

$y_\infty = 0.00381$ m = 0.38 cm < 1.03 cm = $l_{eff} / 250$

7.2 Deflection under cracked condition

EN 12602, (A.42)

7.2.1 Short-term deflection

Ratio of modulus of elasticity of reinforcing steel and AAC:

$n = \frac{E_s}{E_{cm}} = \frac{200000}{2250} = 88.9$

Only compression zone of AAC and reinforcement is considered for the calculation of moment of inertia.

x-equilibrium (height of compression zone from top of the lintel):

$x = \frac{\sqrt{1+4 \cdot d \cdot A}-1}{2 \cdot A} = \frac{\sqrt{1+4 \cdot 207 \cdot 0.00818}-1}{2 \cdot 0.00818} = 109.3 \text{ mm} = 10.93 \text{ cm}$

$A = b \cdot E_{cm} / (2 \cdot A_{s1} \cdot E_S) = 0.00818$

Moment of area of compression zone AAC and reinforcement:

$I_{c,brutto} = \frac{b \cdot x^3}{12} + n \cdot (5 \cdot \pi \cdot (\emptyset_1/2)^4/4 + 5 \cdot \pi \cdot (\emptyset_2/2)^4/4) = 3984.2 \text{ cm}^4$

Parts of moment of inertia from consideration of the reinforcement:

$A_{s1} = 2.51 \text{ cm}^2$, $A_{s2} = 1.01 \text{ cm}^2$

Centre of gravity:

$y_S = \frac{b \cdot x \cdot (h-x/2)+n \cdot (A_{s1} \cdot y_{s1}+A_{s2} \cdot y_{s2})}{b \cdot x+n \cdot (A_{s1}+A_{s2})} = \frac{10719.3}{711.9} = 15.06 \text{ cm}$

y_{s1} and y_{s2} are the distances from the centre of the reinforcement steel to the bottom surface of the lintel.

$I_{ST} = b \cdot x \cdot (h - x/2 - y_s)^2 + n \cdot (A_{s1} \cdot (y_{s1} - y_s)^2 + A_{s2} \cdot (y_{s2} - y_s)^2)$
$= 36.5 \cdot 10.93 \cdot (25.0 - 10.93/2 - 15.06)^2 + 88.9 \cdot (2.51 \cdot (4.3 - 15.06)^2 +$
$1.01 \cdot (21.9 - 15.0)^2) = 38024.5 \text{ cm}^4$

$E_{cm} \cdot I_{ci} = E_{cm} \cdot (I_{C;BRUTTO} + I_{st}) = 2250 \cdot (3984.2 + 38024.5) \cdot 10^{-8} =$
0.945 MN/m^2

Deflection due to load combination 2 (frequent action combinations):

$y_{el} = \frac{5}{48} \cdot \frac{M_{Sd2} \cdot {l_{eff}}^2}{E_{cm} \cdot I_{ci}} = \frac{5}{48} \cdot \frac{0.00691 \cdot 2.567^2}{0.945} = 0.00502 \text{ m}$

$y_{el} = 0.00502 \text{ m} = 0.50 \text{ cm} < 1.03 \text{ cm} = l_{eff} / 250$

7.2.2 Long term deflection

Effective modulus of elasticity:

$E_{c,eff} = E_{cm} / (1 + \varphi) = 2250 / (1 + 1) = 1125 \text{ N/mm}^2$

Ratio of modulus of elasticity of reinforcing steel and AAC:

$n = \frac{E_s}{E_{c,eff}} = \frac{200000}{1125} = 177.8$

Moment of area of AAC and reinforcement:

$I_{c,brutto} = \frac{b \cdot x^3}{12} + n \cdot (5 \cdot \pi \cdot (\text{ø}_1/2)^4 / 4 + 2 \cdot \pi \cdot (\text{ø}_2/2)^4 / 4) = 3996.7 \text{ cm}^4$

Centre of gravity:

$$y_S = \frac{b \cdot x \cdot (h - x/2) + n \cdot (A_{s1} \cdot y_{s1} + A_{s2} \cdot y_{s2})}{b \cdot x + n \cdot (A_{s1} + A_{s2})} = \frac{13645.1}{1024.8} = 13.31 \text{ cm}$$

Moment of inertia for reinforcement:

$I_{ST} = b \cdot x \cdot (h - x/2 - y_s)^2 + n \cdot (A_{s1} \cdot (y_{s1} - y_s)^2 + A_{s2} \cdot (y_{s2} - y_s)^2) = 36.5 \cdot 10.93 \cdot (25.0 - 10.93/2 - 13.31)^2 + 177.8 \cdot (2.51 \cdot (4.3 - 13.31)^2 + 1.01 \cdot (21.9 - 13.3)^2) = 64939.0 \text{ cm}^4$

$$E_{c,eff} \cdot I_{ci} = E_{c,eff} \cdot (I_{C;BRUTTO} + I_{st}) = 1125 \cdot (3996.7 + 64939.0) \cdot 10^{-8} = 0.776 \text{ MN/m}^2$$

Deflection due to load combination 3 (quasi-permanent combinations):

$$y_\infty = \frac{5}{48} \cdot \frac{M_{Sd3} \cdot {l_{eff}}^2}{E_{c,eff} \cdot I_{ci}} = \frac{5}{48} \cdot \frac{0.00568 \cdot 2.567^2}{0.776} = 0.00502 \text{ m}$$

$y_\infty = 0.00502 \text{ m} = 0.50 \text{ cm} < 1.03 \text{ cm} = l_{eff} / 250$

7.3 Combination of deflection uncracked / cracked

EN 12602 (A.44)

7.3.1 Short-term deflection

$$k \cdot p_{II} + (1 - k) \cdot p_I = 0.771 \cdot 0.50 + (1 - 0.771) \cdot 0.31 = 0.46 \text{ cm}$$

With: $k = 1 - 0.8 \cdot \left(\frac{M_{cr}}{M_{sd2}}\right)^2 = 1 - 0.8 \cdot \left(\frac{3.70}{6.91}\right)^2 = 0.771$

M_{cr} cracking moment
M_{sd2} bending moment for frequent combination of loading
p_{II} short-term deflection for cracked condition
p_I short-term deflection for uncracked condition

$y_{el} = 0.46 \text{ cm} < 1.03 \text{ cm} = l_{eff} / 250$

7.3.2 Long-term deflection

$$k \cdot p_{II} + (1 - k) \cdot p_I = 0.661 \cdot 0.50 + (1 - 0.661) \cdot 0.38 = 0.46 \text{ cm}$$

with $k = 1 - 0.8 \cdot \left(\frac{M_{cr}}{M_{sd3}}\right)^2 = 1 - 0.8 \cdot \left(\frac{3.70}{5.68}\right)^2 = 0.661$

M_{cr} cracking moment
M_{sd3} bending moment for frequent combination of loading
p_{II} long-term deflection for cracked condition
p_I long-term deflection for uncracked condition

$y_\infty = 0.46 \text{ cm} < 1.03 \text{ cm} = l_{eff} / 250$